2nd IEEE Int. Conf. on Image Processing,
October 1995, Arlington, USA

DETERMINATION OF SINGULAR POINTS IN 2D DEFORMABLE FLOW FIELDS

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ABSTRACT

Digital image analysis appears to be more and more relevant to the study of physical phenomena involving fluid motion, and of their evolution over time. In that context, 2D deformable motion analysis is one of the important issues to be investigated. The interpretation of such deformable 2D flow fields can generally be stated as the characterization of linear models provided that first order approximations are considered in an adequate neighborhood of so-called singular points, where the velocity becomes null. This paper describes an efficient method, based on a statistical approach, which explicitly addresses these problems, and allows us to locate, characterize and track such singular points in an image sequence. It does not require the prior computation of the velocity field. The method has been validated by experiments carried out with synthetic and real examples corresponding to meteorological image sequences. In fact, the described approach can be of interest in different applications dealing with the characterization of vector fields.

1. INTRODUCTION

Digital analysis of image sequences appears to be more and more relevant to the study of physical phenomena involving fluid motion. This may concern different domains like meteorology, oceanography, or fluid mechanics. In that context, 2D deformable motion analysis is one of the most important issues to be investigated [1, 2, 3]. The interpretation of such deformable 2D flow fields can generally be stated as the characterization of linear models provided that first order approximations are considered in an adequate neighborhood of so-called singular (or critical) points, where the velocity becomes null [4]. However, locating such points, delimiting this neighborhood, and estimating the associated 2D affine flow field, are intricate difficult problems. This paper describes an efficient method based on a statistical approach which explicitly addresses these three problems, and allows us to locate, characterize and track such singular points in an image sequence, using an adaptive on-line determination of the optimal estimation window in position and size.

2. MOTION MODELING AND CLASSIFICATION

We consider a 2D affine motion field in a 2D image plane. It can be described by a first order differential equation:

$$\dot{X} = AX + b \tag{1}$$

where $\dot{X}$ denotes the velocity vector at point X in the image. If the matrix A is not singular, the corresponding motion field has only one singular point $\tilde{X}$ given by:

$$\tilde{X} = -A^{-1}b. \tag{2}$$

The qualitative characterization of the motion field relies on the orientation of the velocity vectors (i.e. on the structure of A)[4]. Six typical motion configurations can be identified (node, saddle, star-node, improper node, center and spiral)[1, 2].

The linear approximation can be considered due to the Gorbman-Hartman theorem [5]. In the neighborhood of a critical point, the linearized velocity field is homeomorphic to the original velocity field (if the latter is continuously differentiable). In others terms, there exist a neighborhood around a critical point where the true motion can be described properly by a linear motion (this means that the two motions share the same structural properties).

We have defined the following scheme to achieved this classification. The knowledge of the sign or the nullity of three functions of the estimated coefficients of matrix A is in fact sufficient to build a decision binary tree which enables to determine the class of the estimated motion [6]. These three functions are the determinant, the trace, and the discriminant of A. Their variances can be easily computed knowing the variances of coefficients of matrix $\hat{A}$. A value f taken by a function F is considered to be effectively null if zero belongs to the interval $[f - \frac{1}{2}\hat{\sigma}_F, f + \frac{1}{2}\hat{\sigma}_F]$, where $\hat{\sigma}_F^2$ is the estimated variance of F. Similarly, the value of F is considered to be positive (resp. negative) if $[f - \frac{1}{2}\hat{\sigma}_F, f + \frac{1}{2}\hat{\sigma}_F]$ is a subset of $\mathbb{R}^+$ (resp. a subset of $\mathbb{R}^-$).

3. ESTIMATION OF THE LINEAR MOTION MODEL

The 2D linear (or more exactly affine) motion model is computed directly from the image intensities using a gradient-based multi-resolution robust estimation method described in [7], using the usual constant brightness assumption [8]. This method takes advantage of a multiresolution framework and an incremental scheme based on the Gauss-Newton technique. Moreover, it minimizes an M-estimator criterion based on a hard-redescending function to ensure the goal of robustness. This estimator allows us to get an accurate estimation of the dominant motion in the considered estimation support, which is of key interest for the estimation of the corresponding singular point (relation (2)), especially at the initial iterations of the algorithm. This method permits also to estimate the variances of the motion parameters [7].

4. ESTIMATION OF CRITICAL POINTS

We want to properly estimate the affine motion model, by finding the neighborhood in which this first order approximation holds in order to correctly locate the corresponding singular point. Moreover, the window center should be as close as possible to the unknown location of the critical point to ensure a reliable estimation. We propose a statistical approach which allows us to handle simultaneously these questions.

We start from a window W_1 which can be taken at random and may or may not include a critical point. Let us now assume that we have performed the i^{th} iteration of the algorithm. That is, we have i-times shifted the estimation window from this initial position, and we have defined the estimation window W_i, of center C_i and size S_i. We have now to define the position C_{i+1} of the next estimation window W_{i+1} and its size S_{i+1}. In the current window W_i, the estimate $\tilde{X}_i$ of the critical point is computed from the estimates $\hat{A}_i$ and $\hat{b}_i$ using relation (2). If $\tilde{X}_i$ lies inside the window W_i, then, we take $C_{i+1} = \tilde{X}_i$. Otherwise, C_{i+1} is given by the intersection between the segment $[C_i, \tilde{X}_i]$ and the border of the window W_i. This somewhat "cautious" way of taking into account the estimation of $\tilde{X}_i$ is due to the fact that we cannot ensure both the linearity of the motion field within W_i and the accuracy of the estimate $\tilde{X}_i$ if $\tilde{X}_i$ lies outside W_i.

If the estimated location of the critical point is outside the window W_i, then the window size does not change ($S_{i+1} = S_i$). Otherwise, the size S_{i+1} of the estimation window may be seen as a nuisance parameter in the estimation of the hidden variable corresponding to the critical point position $\tilde{X}$. The evaluation of the nuisance parameter is based on a risk function $R(\tilde{X}, \tilde{X}_{i+1})$. This function may be decomposed in two terms, bias and variance:

$$R(\tilde{X}, \tilde{X}_{i+1}) = E\|\tilde{X} - \tilde{X}_{i+1}\|^2 = B^2(\tilde{X}_{i+1}) + \hat{\sigma}^2(\tilde{X}_{i+1}). \quad (3)$$

It is rather intuitive that the window size must be sufficiently large to correctly estimate the critical point $\tilde{X}$ in presence of noise (i.e. we want to minimize the estimate variance), and not too large in order to ensure that the linear approximation still holds (i.e. we do not want to introduce bias in the estimation of matrix A, and then, of $\tilde{X}$). Thus, we make the assumption that the bias is an increasing function of the estimation window size S and the variance is a decreasing function of S.

Then, we search the value S^*_{i+1} of S_{i+1} where the bias and the estimate variance are equal. The corresponding critical point is denoted $\tilde{X}^*_{i+1}$.

We will restrict ourselves to a "discrete" version of this problem, and will search the optimal value $\hat{S}_{i+1}$ within a set of n given sizes, $\{S^l_{i+1}, 0 \leq l \leq n-1\}$, with $S^l_{i+1} \leq S^{l+1}_{i+1}, \forall l$. $\tilde{X}^l_{i+1}$ will denote the critical point estimated within the window W^l_{i+1} of size S^l_{i+1}. All the windows W^l_{i+1} have the same center C_{i+1}.

In order to find $\hat{S}_{i+1}$, we adopt a statistical criterion, which has been described in [9]. The best estimator $\hat{S}_{i+1}$ of S^*_{i+1} is the value $S^{\hat{l}}_{i+1}$ which satisfies:

$$\hat{l} = \max\{l : \forall r, r < l : \|\tilde{X}^l_{i+1} - \tilde{X}^r_{i+1}\|^2 \leq 8 * \hat{\sigma}^2(\tilde{X}^r_{i+1})\}. \quad (4)$$

We can give the following intuitive explanation of this test. As long as successive estimates $\tilde{X}^l_{i+1}$ stay close to each other, we decide that the bias (3) is "small" and the size of the estimation window can be increased to improve the estimation of the linear model. If an estimated point $\tilde{X}^l_{i+1}$ appears "far" from the precedent ones, we interpret this as the dominance of the bias over the variance term (the motion field cannot be considered anymore linear in the window W^l_{i+1}). The theoretical derivation of this test is beyond the scope of the paper (see [9] and [10] for details). Convergence is reached when the estimation window does not need to be changed in position, nor in size.

5. RESULTS

In order to find all critical point locations, we scan the image by considering overlapping windows, i.e. square blocks. Our estimation method allows us to consider a not too dense subdivision of the image into blocks, thus saving CPU time while bringing more robustness. Shifts in rows and columns are such that two neighboring blocks overlap each other by the half of their size. Moreover, given an initial block, the algorithm is run if the following condition is satisfied. At the first iteration, the distance between the estimation window W_1 and the corresponding estimated singular point $\tilde{X}_1$ must be lower than a given threshold (typically, the width of the block).

Tracking over time of the singular points proceeds as follows. Starting with the critical points detected in the first frame, we estimate their new location using only one initial block for each point in the second image. It is given by the final estimation window in the previous frame. This may be repeated for all the sequence frames. Moreover, we can also eliminate spurious critical points by checking the temporal coherence of their detection over time.

We have applied our method on several image sequences. We report here results obtained on two real sequences. The first one (Fig.1) is a sequence of meteorological pictures depicting a large scale atmospheric disturbance on the western Europe and the north of Africa. The second one (Fig.2) is another real image sequence taken by Meteosat. It also contains an atmospheric disturbance. Results, which validate our approach, are commented upon in the captions of Fig. 1 and 2. Other experiments can also be found in [6].

6. CONCLUSION

We have developed a new method to detect and locate singular points in a deformable flow field. Besides, it permits to qualitatively interpret its structure while taking into account the uncertainty in the estimation of the motion parameters. It must be pointed out that our method does not depend strongly on the initialization. It also appears to be more efficient and reliable than a vote technique combining estimates systematically computed in a set of blocks. Furthermore, the principle of our approach allows us to use the proposed method not only to detect singular point in a given image, but also, without modification, to track them along the sequence. Several experiments (with synthetic and real motions) have validated this approach. Applications of such a technique are numerous and important.

7. REFERENCES

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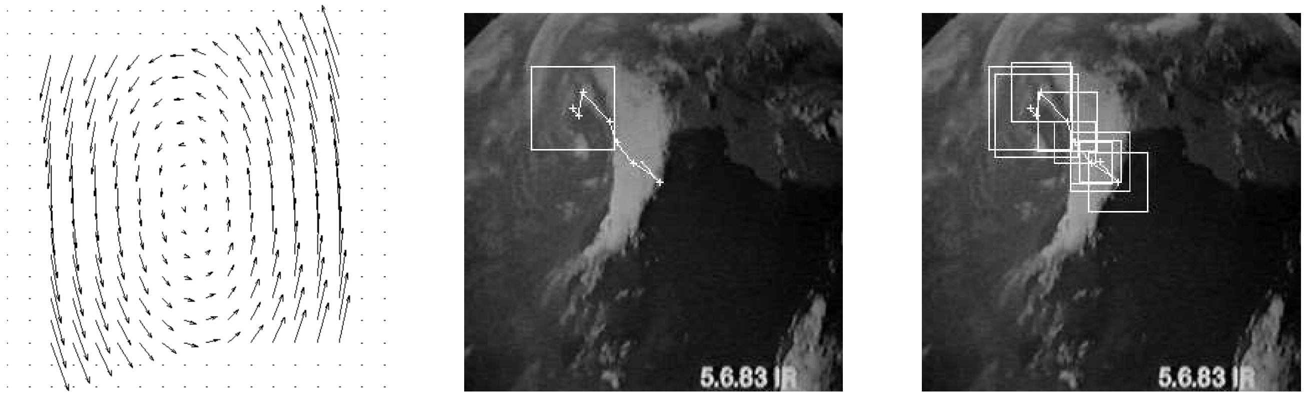
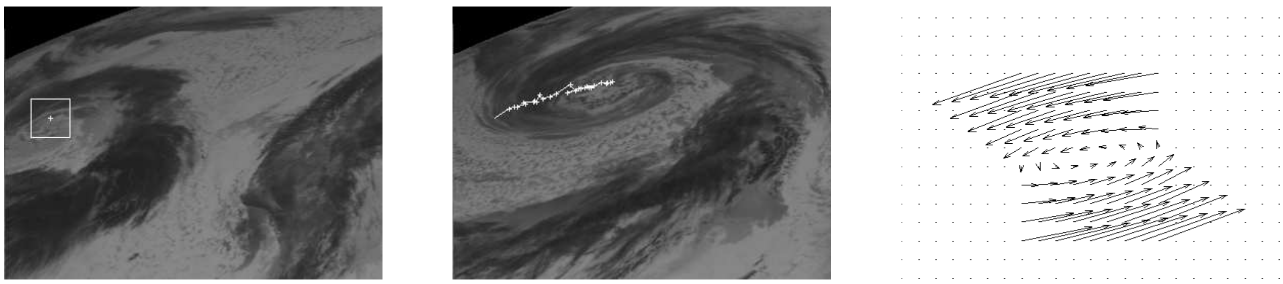


(a) (b) (c)

Figure 1: First real meteorological sequence, a disturbance. (a) The linear flow subfield estimated around the critical point, well classified as a spiral one, is plotted in the final estimation window (zoomed), one every 5 vector is displayed, and the vector magnitudes are magnified by a factor of 4. (b) Successive positions of the estimation window center along the algorithm iterations, the initialisation is near the image center. This path converges to the spiral point. The final estimation window (white rectangular) is also plotted. (c) The successive estimation windows are plotted (the initialisation is the same as in (b)).



(a) (b) (c)

Figure 2: Second real meteorological image sequence (30 images). (a) The first frame of the sequence; the validated critical point is pointed with a white cross and the final estimation window is visualized with a white rectangular. (b) The estimated uncertainties of $\tilde{X}$ location is about 7 pixels in x and y. The tracking over the time of this critical point is superimposed on the last frame of the image sequence. We can note that the estimated position of the spiral critical point, in the last image, correctly coincides with the real data. (c) The estimated sub-field of the estimated critical point is plotted in the last estimation window, one every 4 vector is displayed, and the vector magnitudes are magnified by a factor of 2.

Acknowledgments

Meteorological image sequences (Fig. 1 and 2) have been respectively provided by CMS, Lannion, France and LMD, Palaiseau, France.