Structured Sparsity through reweighting and Application to diffusion MRI

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Introduction and Outline

Problem: Recovery of ***multiple correlated sparse*** signals

Outline:

✓ Diffusion MRI and problem formulation

✓ Structured sparsity through reweighting

✓ Results

✓ Discussion and future work

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Diffusion MRI (dMRI)

✤ What is it?

✓ Diffusion MRI measures the Brownian motion of water molecules in a fluid due to thermal energy.
✓ In ordered tissues, water does not diffuse equally in all directions (*anisotropic* diffusion).

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Study the spatial order in living organs in a non-invasive way.

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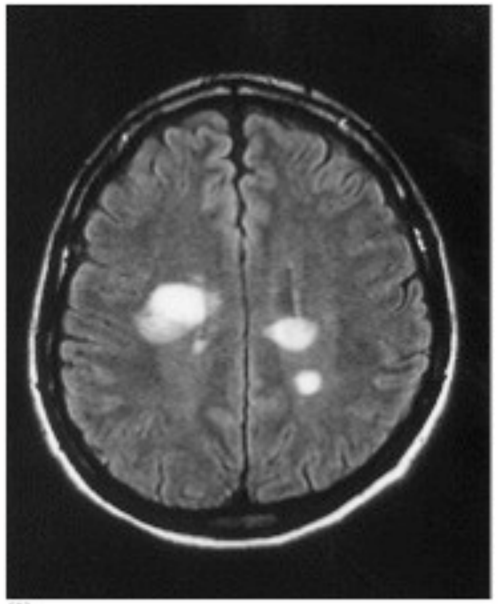

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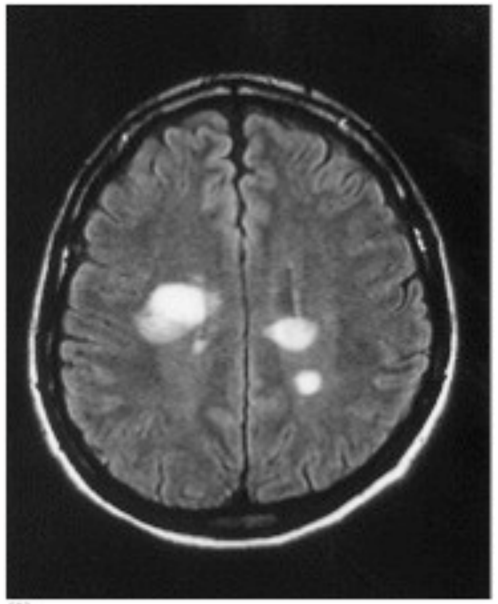

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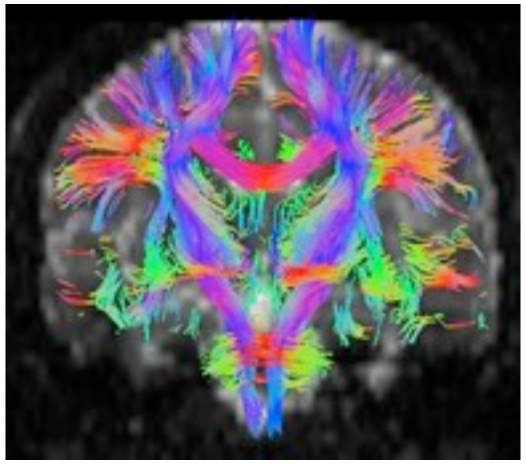

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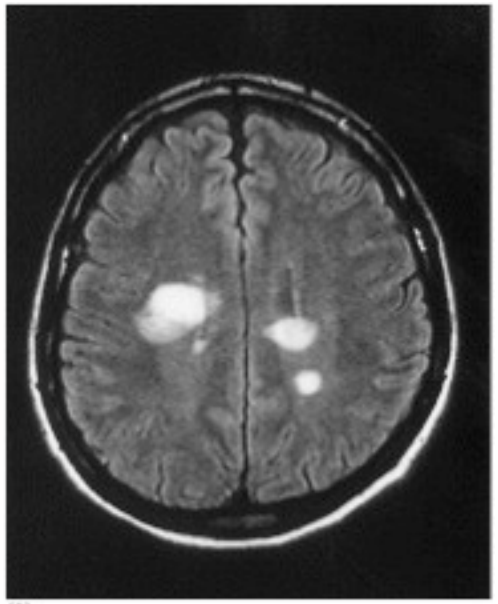
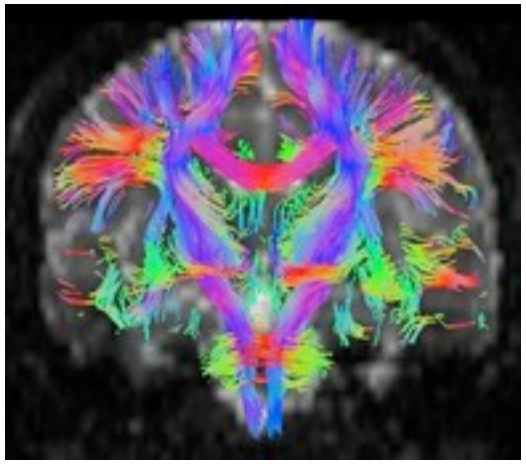


(A)

✤ STRUCTURAL NEURAL CONNECTIVITY

✓**Why?** Neuroscience / Clinical applications
✓**How?** Fiber tracking (tractography)



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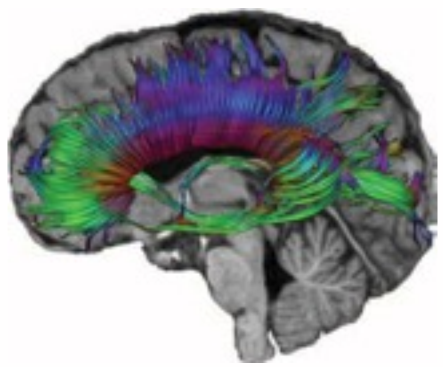
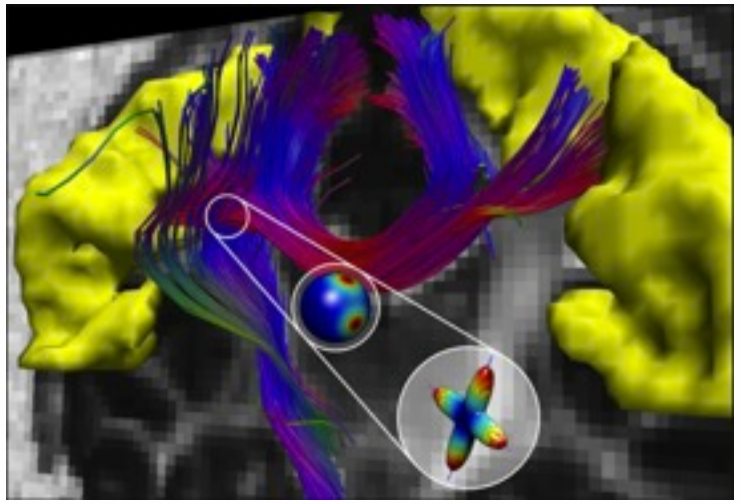
dMRI: Local Reconstruction problem





Recover the ***fiber orientation*** in every voxel of the brain.

Function of interest:

✓ Fiber Orientation Distribution (FOD)

Probability of having a fiber along a given direction (function on S^2)

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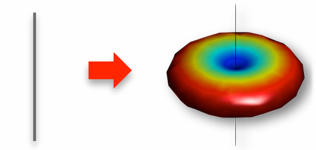
dMRI: FOD recovery via sparse reconstruction

Assumptions:

1. Diffusion characteristics of all fiber in the brain are identical.



✓ KERNEL: Response generated by a single fiber estimated from the data.

2. No exchange between spatially distinct fiber bundles.

$$f_1 S_1(\theta,\phi) + f_2 S_2(\theta,\phi) = S(\theta,\phi) = R(\theta) \otimes F(\theta,\phi)$$

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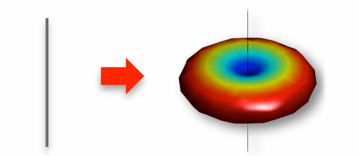

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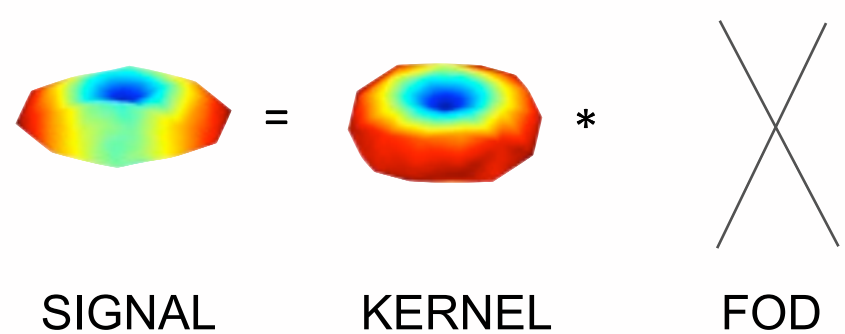
Spherical Deconvolution methods assume the signal can be expressed as the convolution of a kernel with the Fiber Orientation Distribution:



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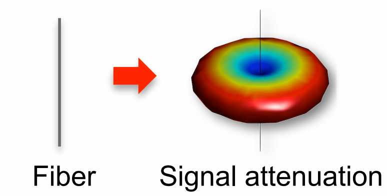

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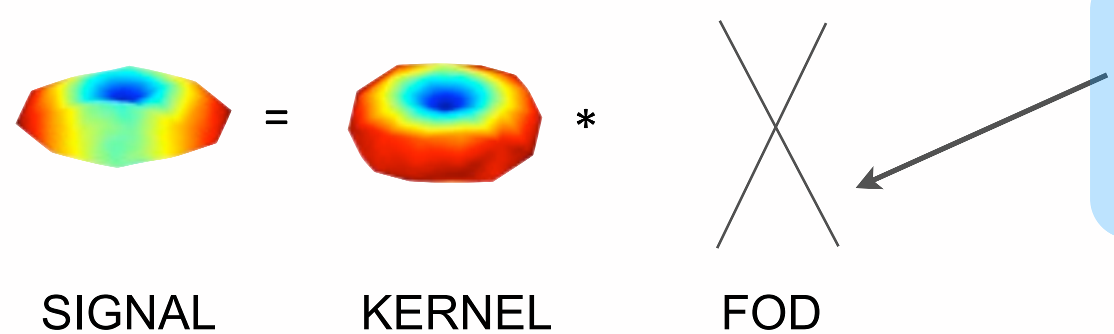


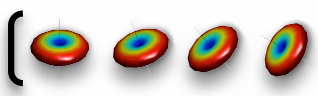
✓ non-negative

✓ sparse

dMRI: FOD recovery via sparse reconstruction

The intra-voxel recovery problem can be expressed **voxelwise** in terms of the following linear formulation: (Jian and Vemuri, 2007)

$$y = \Phi x + \eta$$

- ✓ y is the acquired diffusion MRI data and x is the FOD (of a single voxel).
- ✓ Φ is the *sensing basis* or *dictionary.*  ← Each atom of the dictionary is associated to a discrete direction on the sphere
- ✓ η represents de acquisition noise.

Reweighted constrained ℓ_1 minimization (Candes et al, 2008)

$$\min_{x \geq 0} \quad \|\Phi x - y\|_2^2 \quad \text{s.t.} \quad \|x\|_{w,1} \leq k$$

sparsity term

where

$$\|x\|_{w,1} = \sum_i w_i |x_i|$$

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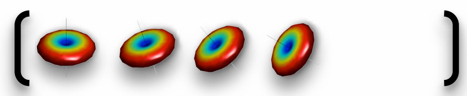
- ✓ Φ is the *sensing basis* or *dictionary*.  ← Each atom of the dictionary is associated to a discrete direction on the sphere
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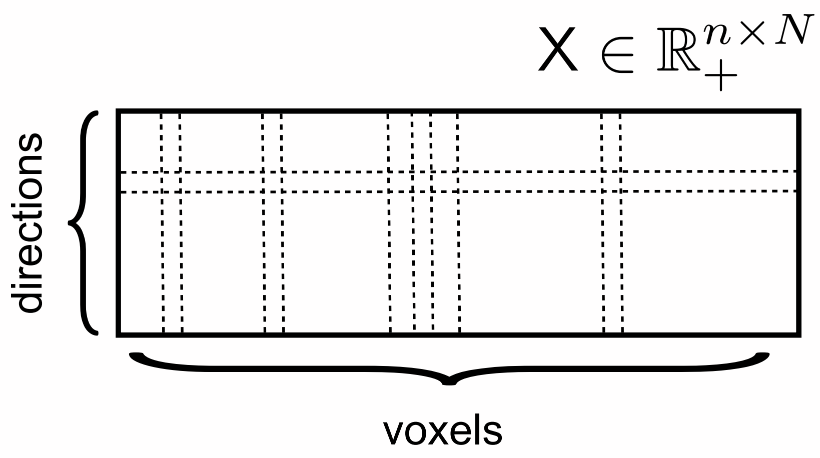
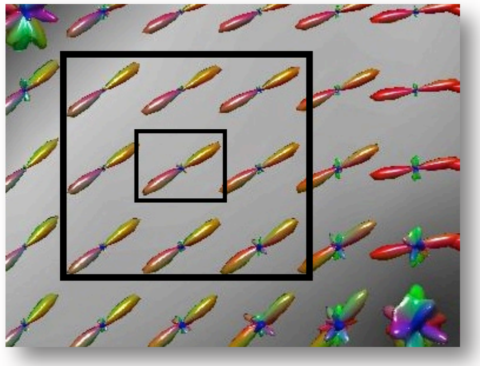
Solving a sequence of these weighted problems with $w_i^{(t)} \approx 1/x_i^{(t-1)}$

Structured Sparsity through reweighting in dMRI

✓**IDEA**: Solve the FOD field for all voxels simultaneously to exploit spatial coherence between neighboring voxels:





✓**Assumption**: neighbor voxels should present the same/neighbor directions.

Proposed formulation:

$$\min_{\mathsf{X} \in \mathbb{R}_+^{n \times N}} \|\Phi \mathsf{X} - \mathsf{Y}\|_2^2 \quad \text{s.t.} \quad \|\mathsf{X}\|_{\mathsf{W},1} \leq K.$$

sparsity and structure spatial regularisation

with $\|\mathsf{X}\|_{\mathsf{W},1} = \sum_{d,v} \mathsf{W}_{dv} |\mathsf{X}_{dv}|$

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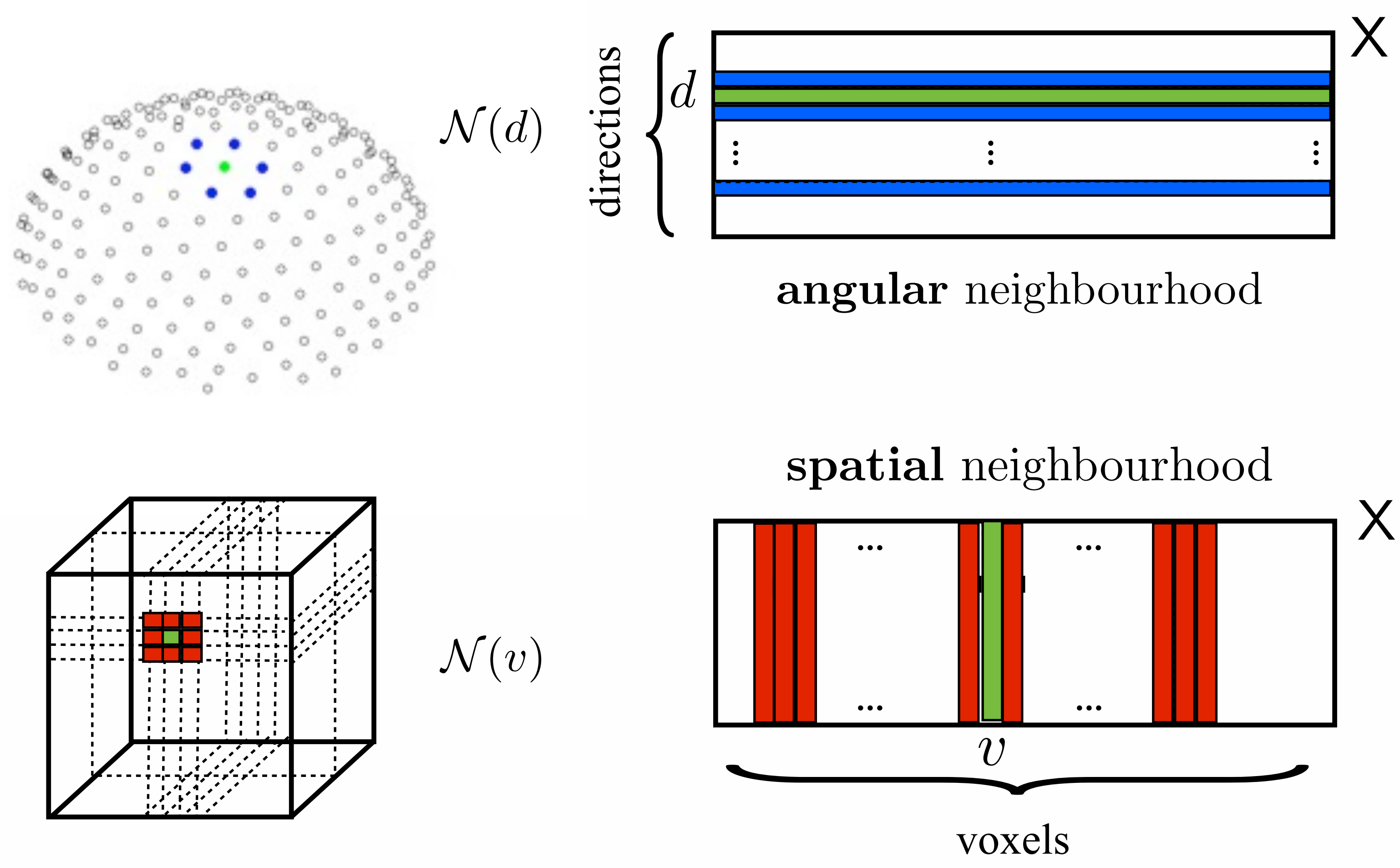
Definition of neighborhood:



Spherical deconvolution: global problem

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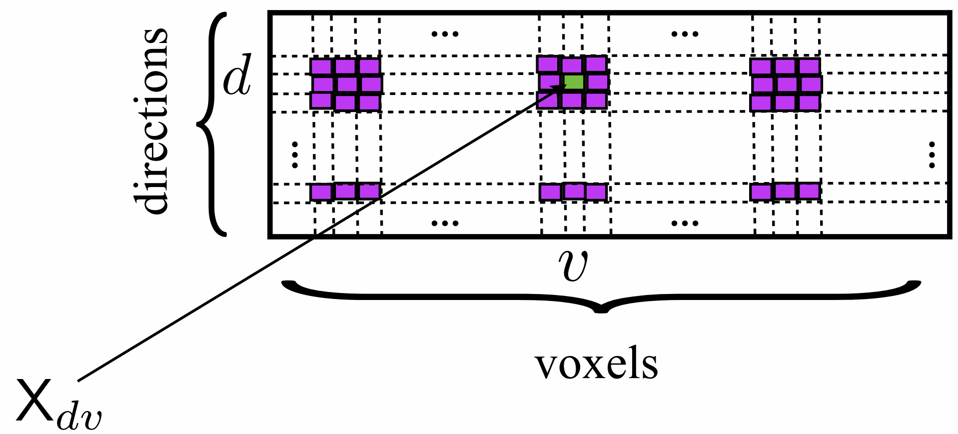
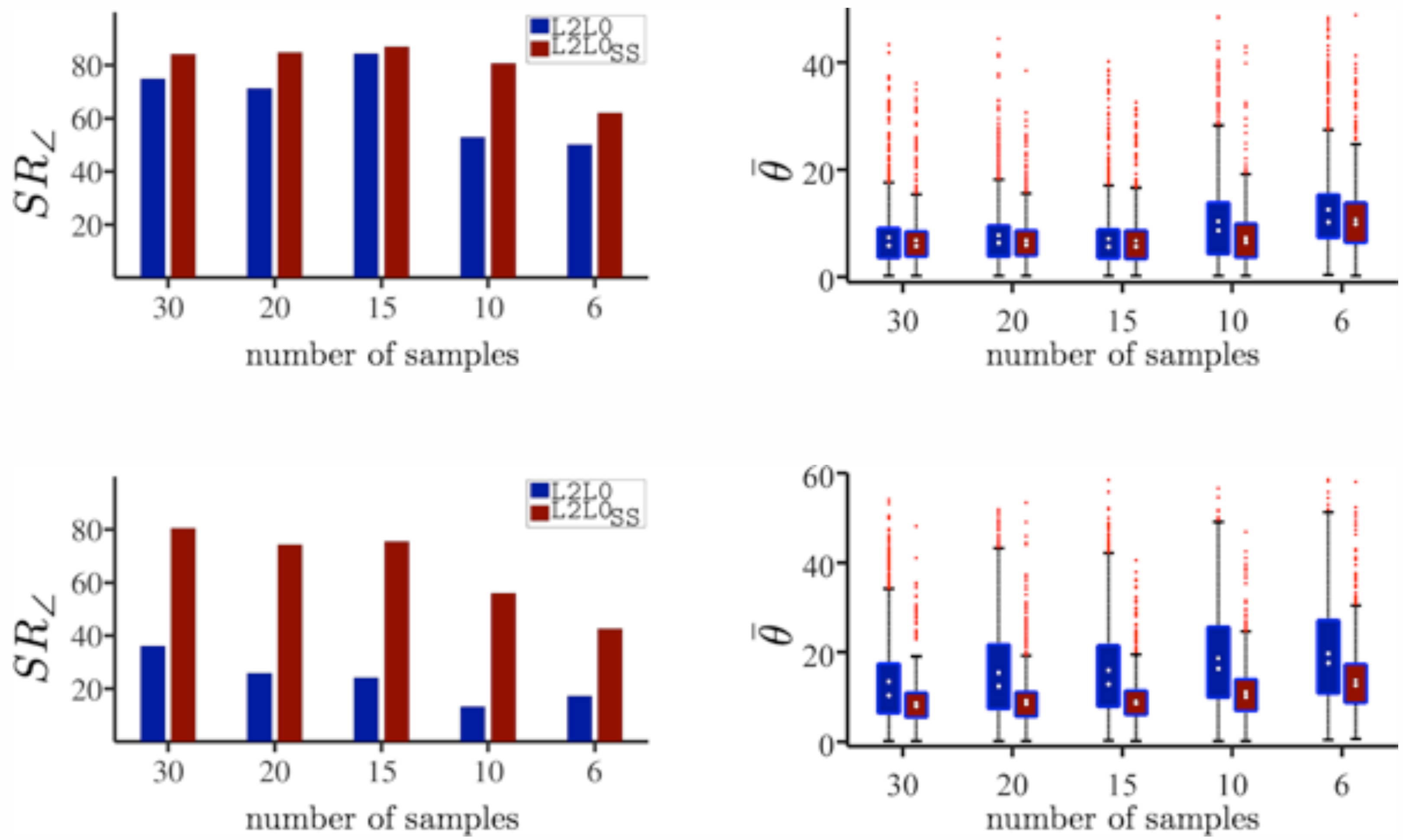


Definition of the weights:

$$W_{dv}^{(t+1)} = \frac{1}{\tau^{(t)} + \frac{\sum_{d'v' \in \mathcal{N}(dv)} |X_{d'v'}^{(t)}|}{|\mathcal{N}(v)|}}$$

Simulations and Results



Exploiting spatial coherence → Undersampling regimes → Speed up acquisition

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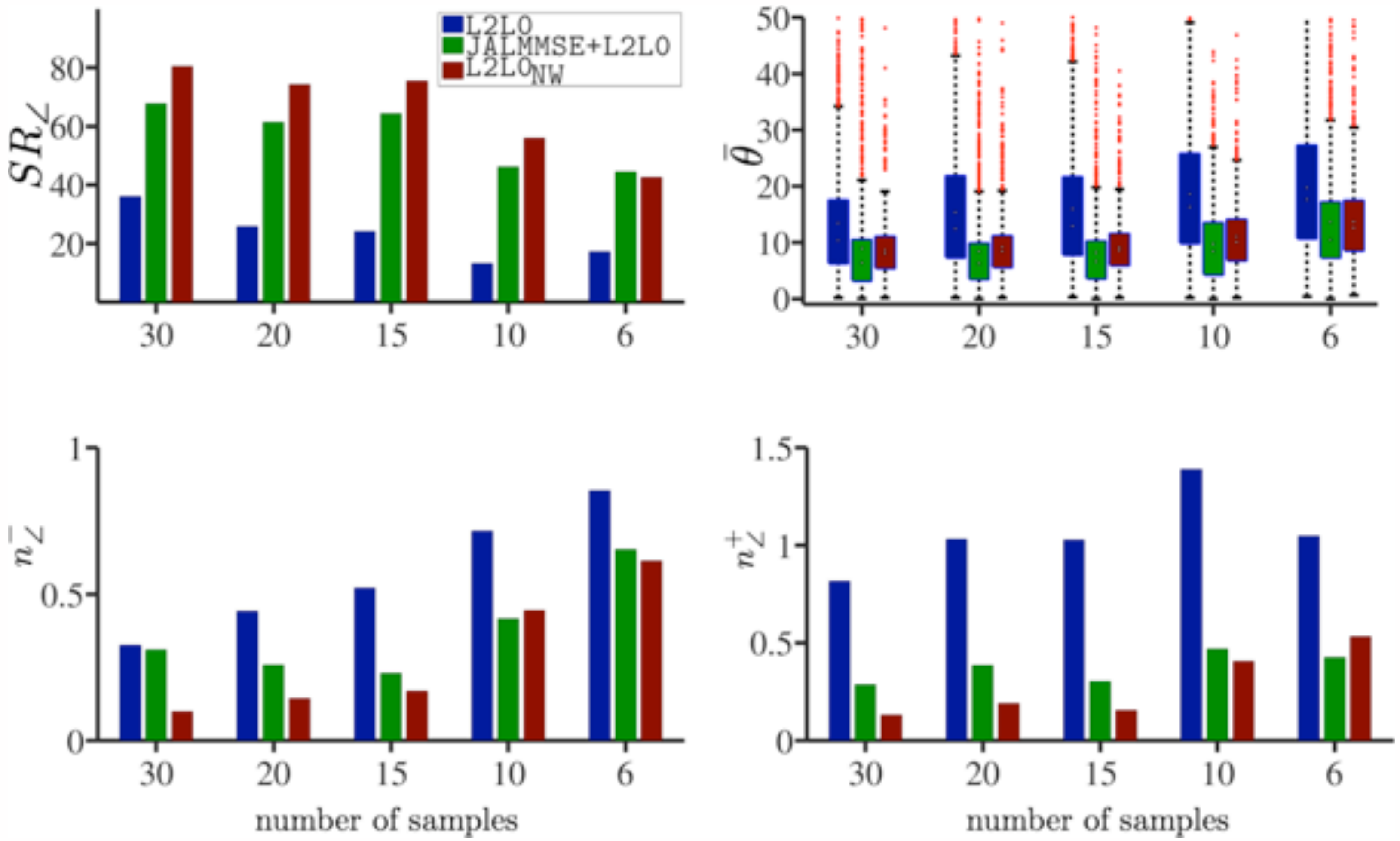

Exploiting spatial coherence → Undersampling regimes → Speed up acquisition

Conclusions

CONCLUSIONS:

✓ Spatially structured sparsity guaranties **robustness to noisy** and ability to go to **higher undersampling** regimes.

✓The method is versatile and can be generalised to recover multiple correlated sparse signals

FUTURE WORK:

✓ in dMRI: application to recovery of microstructure properties of the tissue, tractography methods,...

THANK YOU

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