Human Activity and Vision Summer School

Macroscopic models for crowd behavior simulation

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Outline of the talk

Crowd dynamics

Model to reproduce known pedestrian behavior:

- evacuation dynamics: seeking the *fastest* route, avoiding high densities and borders (discomfort)
- desired speed ($\sim 1.34 m/s$), depending on situations
- lines and more general patterns formation: self-organization dynamics which minimize interactions with the opposite stream
- oscillations at bottlenecks in opposite streams passing through a narrow passage
- collective auto-organization at intersections: increasing the average efficiency
- etc ...

(Helbing-Farkas-Molnar-Vicsek 2002)

Panic

Crowd behavior changes in **panic** situations and becomes irrational:

- getting nervous → "freezing by heating"
- people try to move faster → clogging → "faster is slower"
- jams building up at exits → fatal pressures
- escape slowed down
- herding behavior (to follow others) → ignorance of available exits
- "phantom panics" due to counterflow and impatience

(Helbing-Farkas-Molnar-Vicsek 2002)

21st century main human stampedes

Date	Place	Venue	Deaths	Reason
2001	Accra, Ghana	stadium	126	tear gas
2003	West Warwick, RI	nightclub	100	fire
2004	Mecca, Saudi Arabia	Jamarat Bridge	251	overcrowding
2005	Maharashtra,India	religious procession	265	overcrowding
2005	Baghdad, Iraqi	religious procession	953	rumors of bomb
2006	Mecca, Saudi Arabia	Jamarat Bridge	345	overcrowding
2006	Pasig CityPhilippines	stadium	78	rush for tickets
2008	Himachal Pradesh, India	religious procession	162	panic
2008	Jodhpur, India	religious procession	147	rumors of bomb
2010	Duisburg, Germany	Love Parade	21	mass panic
2010	Phnom Penh, Cambodia	water festival	347	panic
2011	Kerala, India	religious procession	106	car in the crowd

Source: `http://en.wikipedia.org/wiki/Stampede`

Outline of the talk

Pedestrian flow models

Three possible scales:

- Microscopic:
  - social force models, cellular automata, agent-based models...
  - ODEs system
  - numerical simulations (`http://angel.elte.hu/~panic/pedsim/`)
  - many parameters
  - huge literature:

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- Macroscopic: PDEs from fluid dynamics, optimal transport, mean-field games
  - analytical theory
  - few parameters
  - suitable to formulate control and optimization problems
  - more recent research: Bellomo-Dogbé, Bruno-Venuti, Colombo-Rosini, (Henderson), Hughes, Kachroo, Maury, Piccoli, Santambrogio, Treuille-Cooper-Popovič...

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- Kinetic: distribution function of the microscopic states
  - Bellomo-Dogbé, Bellomo-Bellouquid, Carrillo, Degond ...
- Macroscopic: PDEs from fluid dynamics, optimal transport, mean-field games
  - analytical theory
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Macroscopic models

$$\left[\text{number of individuals in } \Omega \subset \mathbb{R}^2 \text{ at time } t\right] = \int_\Omega \rho(t, \mathbf{x})\, d\mathbf{x}$$

must be conserved!

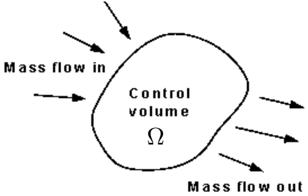
$$\int_\Omega \rho(t_2, \mathbf{x}) d\mathbf{x} = \int_\Omega \rho(t_1, \mathbf{x}) d\mathbf{x} - \int_{t_1}^{t_2} \int_{\partial\Omega} \mathbf{f}(t, \sigma) \cdot \vec{n}\ d\sigma\, dt$$

$$\Downarrow$$

divergence theorem for $(\rho, \mathbf{f})$

$$\Downarrow$$

$$\int_{t_1}^{t_2} \int_a^b \partial_t \rho + \operatorname{div}_{\mathbf{x}} \mathbf{f}\ d\mathbf{x}\ dt = 0$$



Macroscopic models

- Conservation law:

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \mathbf{f}(t\, x) = 0$$

- Flux-density relation: $\mathbf{f}(t, \mathbf{x}) = \rho(t, \mathbf{x}) \vec{V}(t, \mathbf{x})$

- Density must be non-negative and bounded: $0 \leq \rho(t, \mathbf{x}) \leq \rho_{\max}$, $\forall \mathbf{x}, t > 0$ (maximum principle?)

- Different from fluid dynamics:
  - preferred direction
  - no conservation of momentum / energy
  - no viscosity
  - $n \ll 6 \cdot 10^{23}$

Continuum hypothesis

$n \ll 6 \cdot 10^{23}$ but ...

Continuum hypothesis

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Portland, Oregon, May 2008

Outline of the talk

Scalar conservation laws

We deal with a PDE equation of the form

$$\begin{aligned} &\partial_t \rho + \mathrm{div}_{\mathbf{x}} \mathbf{f}(t, \mathbf{x}, \rho) = 0 \\ &\rho(0, \mathbf{x}) = \rho_0(\mathbf{x}) \\ &+\mathrm{BC} \end{aligned}$$

where $t \in [0, +\infty[$, $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$,
$\rho = \rho(t, \mathbf{x}) \in \mathbb{R}$ conserved quantity
$\mathbf{f} : [0, +\infty[\ \times \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ flux

Main features:

- ρ NOT smooth
- existence $\leftarrow$ weak solutions
- uniqueness $\leftarrow$ entropy conditions

Kružkov theory (1970)

- smooth flux: $\mathbf{f} \in \mathcal{C}_c^1\left([0,+\infty[\,\times\mathbb{R}^2\times\mathbb{R}\right)$
- entropy weak solution: $\forall k \in \mathbb{R}$ and $\varphi \in \mathcal{C}_c^1\left([0,+\infty[\,\times\mathbb{R}^2\right)$

$$\int_0^{+\infty}\iint_{\mathbb{R}^2} |\rho - k|\partial_t\varphi + \text{sgn}(\rho - k)(\mathbf{f}_i(t,\mathbf{x},\rho) - \mathbf{f}_i(t,\mathbf{x},k))\partial_{x_i}\varphi$$
$$-\text{sgn}(\rho - k)\partial_{x_i}\mathbf{f}_i\ \varphi\ d\mathbf{x}dt \geq 0$$

- well posedness: existence, uniqueness, stability

Outline of the talk

Eikonal equation

Consider $\Omega \subset \mathbb{R}^2$ walking facility $(\partial\Omega = \partial\Omega_{wall} \cup \partial\Omega_{in} \cup \partial\Omega_{exit})$;
we look for $\phi : \Omega \to \mathbb{R}$ solution of the PDE equation

$$|\nabla_{\mathbf{x}}\phi| = C(\mathbf{x}) \quad \text{in } \Omega$$

$$\phi(t, \mathbf{x}) = 0 \qquad \text{for } \mathbf{x} \in \partial\Omega_{exit}$$

where $C = C(\mathbf{x}) \geq 0$ is the *running cost*:

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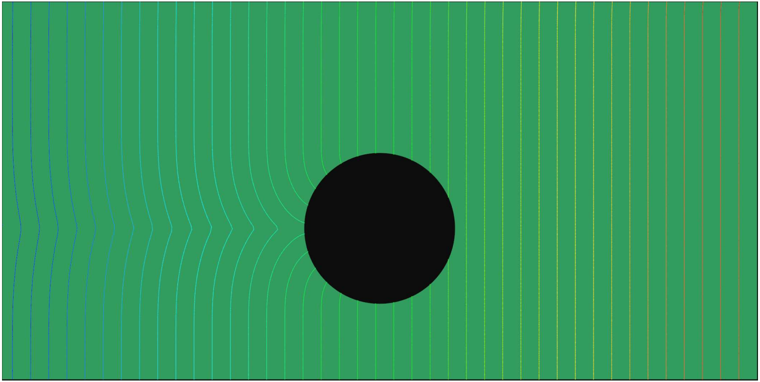
where $C = C(\mathbf{x}) \geq 0$ is the *running cost*:

the solution $\phi(\mathbf{x})$ represents the (weighted) distance of the position $\mathbf{x}$ from the target $\partial\Omega_{exit}$

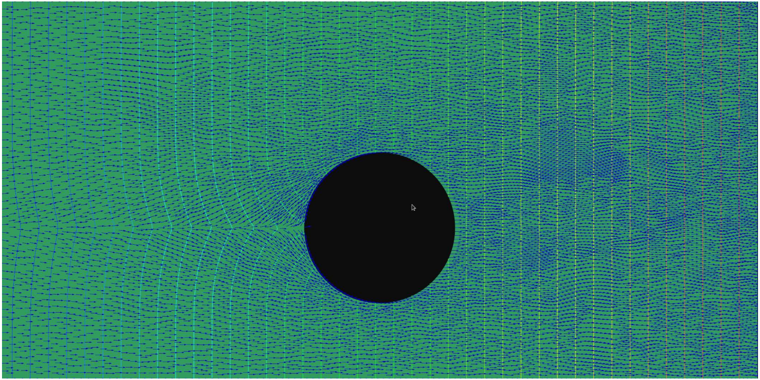
$$\text{if } C(\mathbf{x}) \equiv 1 \text{ and } \Omega \text{ concave then } \phi(\mathbf{x}) = d(\mathbf{x}, \partial\Omega_{exit})$$

(existence and uniqueness under some regularity assumption on C)

Eikonal equation: level set curves for $|\nabla_{\mathbf{x}}\phi| = 1$



Eikonal equation: vector field $\vec{N} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|}$



Outline of the talk

Hughes' model (2002)

Mass conservation

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \left(\rho \vec{V}(\rho) \right) = 0 \quad \text{in } \mathbb{R}^+ \times \Omega$$

where

$$\vec{V}(\rho) = v(\rho)\vec{N} \quad \text{and} \quad v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$$

Direction of the motion: $\vec{N} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|}$ is given by

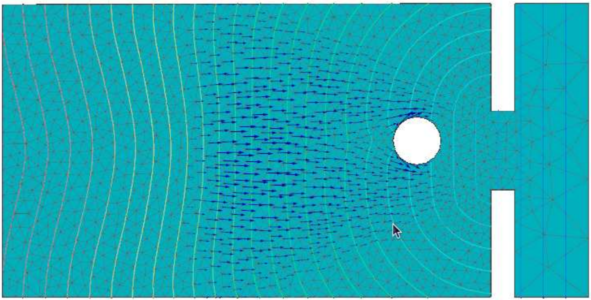
$$|\nabla_{\mathbf{x}}\phi| = \frac{1}{v(\rho)} \quad \text{in } \Omega$$

$$\phi(t, \mathbf{x}) = 0 \qquad \text{for } \mathbf{x} \in \partial\Omega_{exit}, \forall t \geq 0$$

- pedestrians tend to minimize their estimated travel time to the exit
- pedestrians temper their estimated travel time avoiding high densities
- CRITICS: instantaneous global information on entire domain

Hughes' model

Evolution of the velocity field:



$T = 0.2$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

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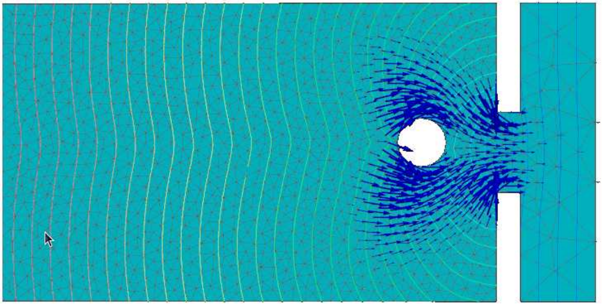

$T = 0.5$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

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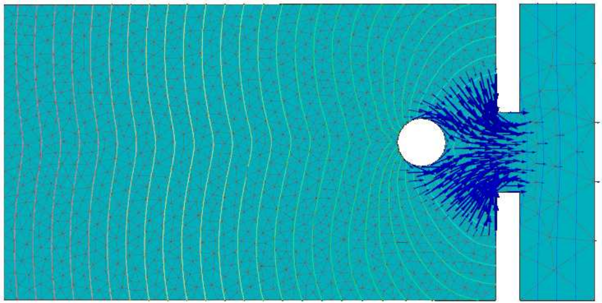
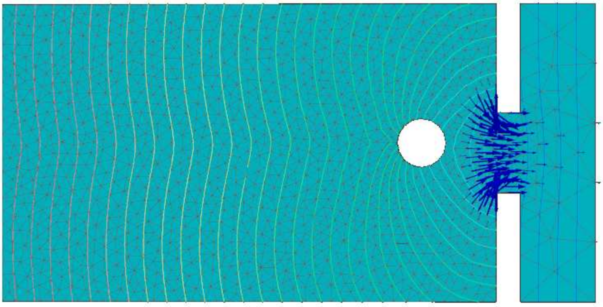


$T = 0.8$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

Evolution of the velocity field:



$T = 1.1$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Dynamic model with memory effect

Mass conservation

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \left(\rho \vec{V}(\rho) \right) = 0 \quad \text{in } \mathbb{R}^+ \times \Omega$$

where

$$\vec{V}(\rho) = v(\rho)\vec{N} \quad \text{and} \quad v(\rho) = v_{\max} \left(1 - \frac{\rho}{\rho_{\max}} \right)$$

Direction of the motion: $\vec{N} = -\dfrac{\nabla_{\mathbf{x}}(\phi + \omega D)}{|\nabla_{\mathbf{x}}(\phi + \omega D)|}$ where

$$|\nabla_{\mathbf{x}} \phi| = \frac{1}{v_{\max}} \quad \text{in } \Omega, \quad \phi(\mathbf{x}) = 0 \text{ for } \mathbf{x} \in \partial\Omega_{exit},$$

$$D = D(\rho) = \frac{1}{v(\rho)} + \beta\rho^2 \quad \text{discomfort}$$

- pedestrians seek to minimize their estimated travel time based on their knowledge of the walking domain
- pedestrians temper their behavior locally to avoid high densities

(Xia-Wong-Shu, 2009)

Second order model

Euler equations with relaxation

$$\partial_t \rho + \nabla \cdot (\rho \vec{V}) = 0$$

$$\partial_t \vec{V} + (\vec{V} \cdot \nabla)\vec{V} + c^2(\rho)\frac{\nabla \rho}{\rho} = \frac{v_e(\rho)\vec{N} - \vec{V}}{\tau}$$

(Jiang-Zhang-Wong-Liu, 2010)

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where

$$v_e(\rho) = v_{\max} \exp\left(-\gamma \left(\frac{\rho}{\rho_{\max}}\right)^2\right), \qquad c(\rho) = c_0 \left(\frac{\rho}{\rho_{\max}}\right)^\beta$$

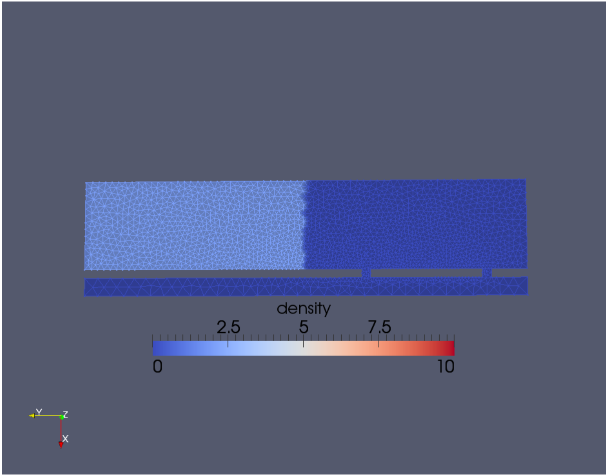
and boundary conditions: $\nabla_{\mathbf{x}} \rho \cdot \vec{n} = 0$ and $\vec{V} \cdot \vec{n} = 0$

(Jiang-Zhang-Wong-Liu, 2010)

Outline of the talk

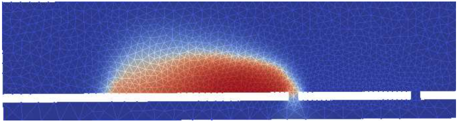
The fastest route ...

... needs not to be the shortest!

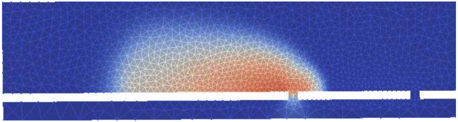


The fastest route ...

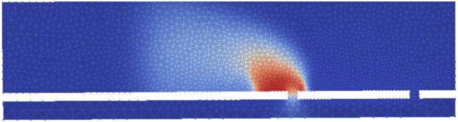
... depends on the model!



$$|\nabla_{\mathbf{x}}\phi| = 1$$



$$\nabla_{\mathbf{x}}(\phi + \omega D)$$



$$|\nabla_{\mathbf{x}}\phi| = 1/v(\rho)$$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

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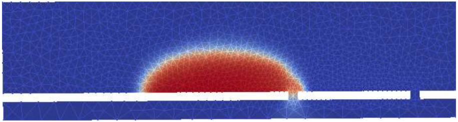

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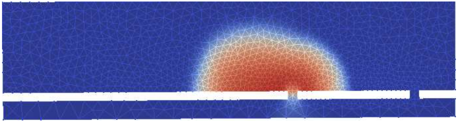

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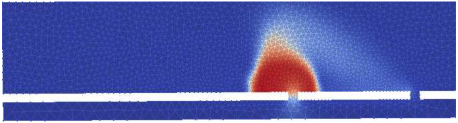

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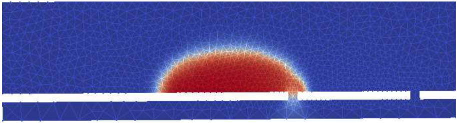

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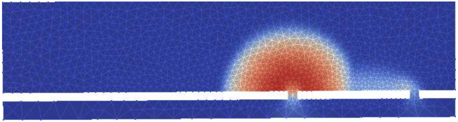

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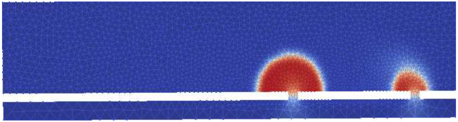

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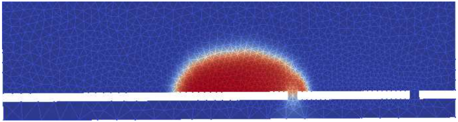

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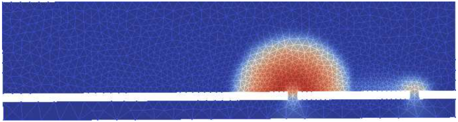

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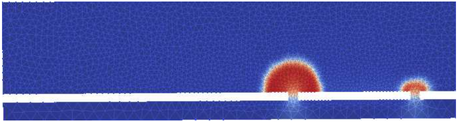

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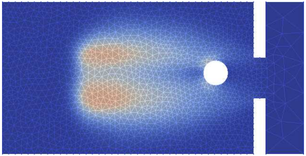
Braess' paradox?

A column in front of the exit can reduce inter-pedestrians pressure and evacuation time?



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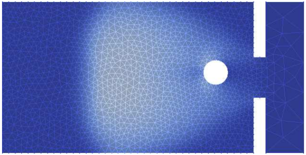

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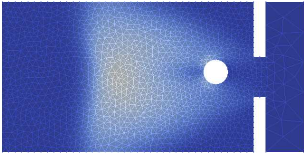

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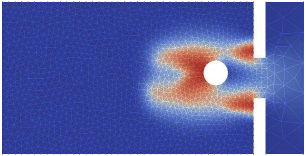

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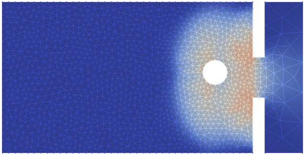

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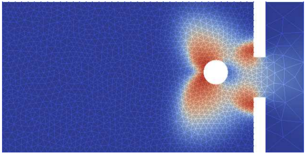

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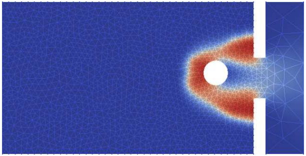

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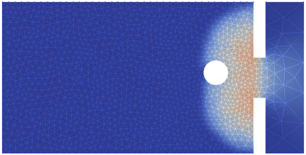

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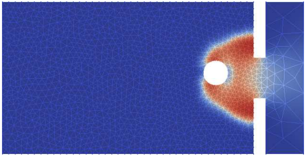

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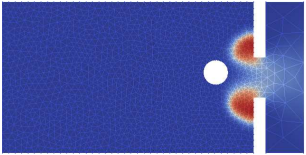

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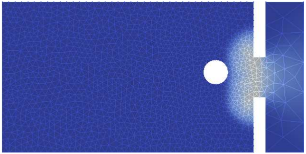

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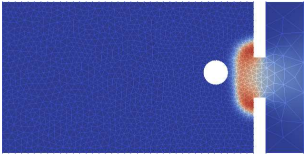

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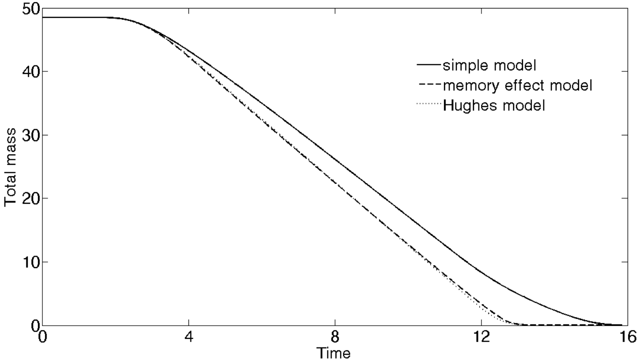
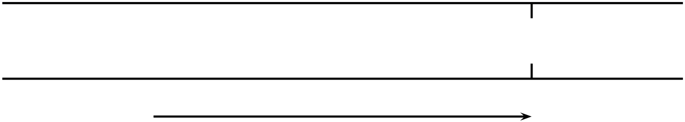
Braess' paradox?

Evacuation time:



(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Panic: a 1D toy model



Mass conservation

$$\partial_t \rho + \partial_x (\rho v(\rho)) = 0$$

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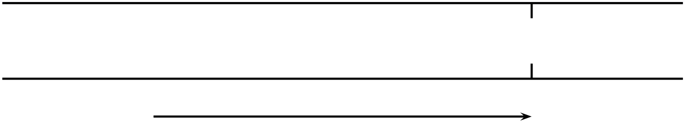

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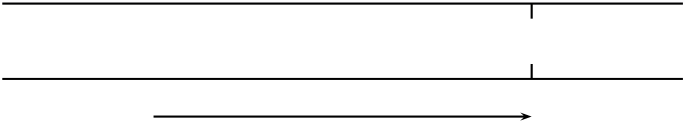

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(Colombo-Rosini, 2005)

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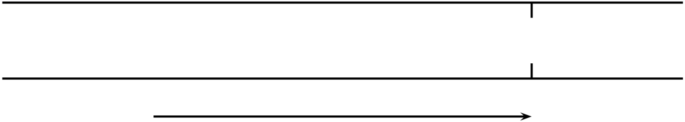

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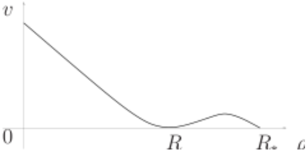

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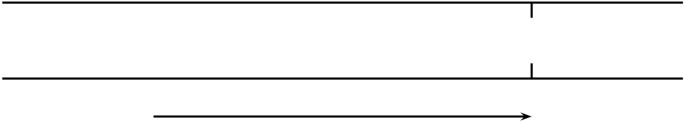

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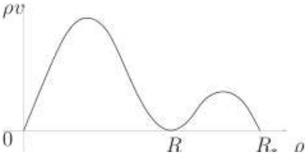

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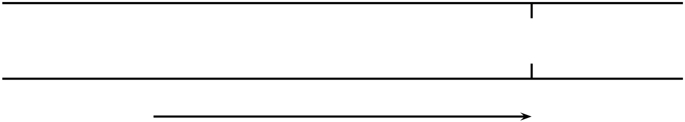

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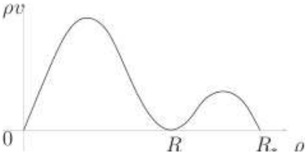
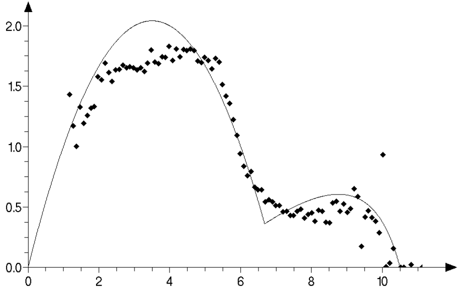


Change the evolution $\to$ non-classical shocks

(Colombo-Rosini, 2005)

Panic: a 1D toy model

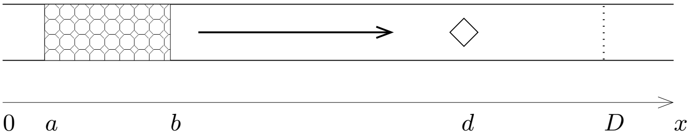
Experimental data:



(Helbing-Johansson-Al Abideen, *Physical Review E*, 2007)

Panic: a 1D toy model

Colombo-Goatin-Rosini '09: **Braess paradox**

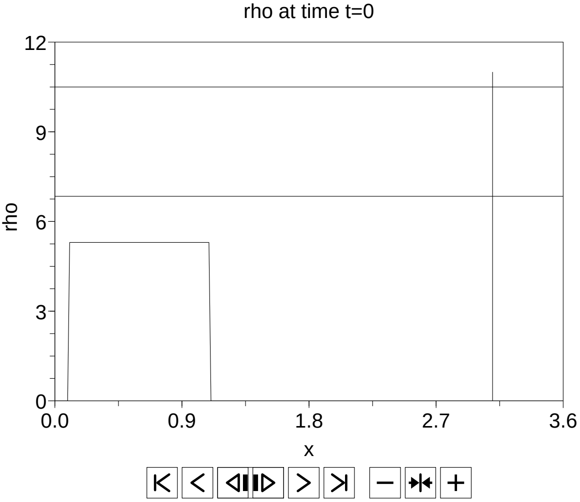


$$\begin{array}{l} \partial_t \rho + \partial_x f(\rho) = 0 \\ \rho(0,x) = \rho_0(x) \end{array} \qquad \begin{array}{rcl} f\left(\rho(t,d)\right) & \leq & q\left(\rho(t,d)\right) \\ f\left(\rho(t,D)\right) & \leq & Q\left(\rho(t,D)\right) \end{array}$$

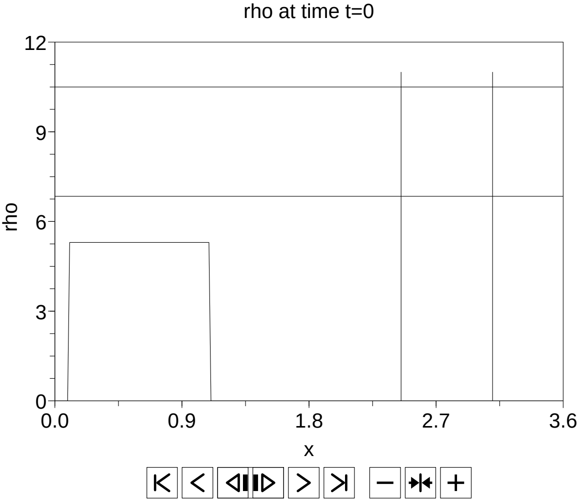
$$q(\rho) = \left\{ \begin{array}{llll} \hat{q} & \text{if} & \rho \in & [0,R] \\ \check{q} & \text{if} & \rho \in &]R,R^*] \end{array} \right. \qquad Q(\rho) = \left\{ \begin{array}{llll} \hat{Q} & \text{if} & \rho \in & [0,R] \\ \check{Q} & \text{if} & \rho \in &]R,R^*] \end{array} \right.$$

$$\hat{q} > \check{q} \qquad \qquad \hat{Q} > \check{Q}$$

Evacuation without obstacle: panic appears



Evacuation with obstacle: no panic



Outline of the talk

Macroscopic models of pedestrians flows

PDEs describing the evolution of macroscopic quantities (e.g. density):

$$\partial_t u(t,\mathbf{x}) + \mathrm{div}_{\mathbf{x}} f(u(t,\mathbf{x})) = 0 \qquad t > 0,\ \mathbf{x} \in \mathbb{R}^D,\ u \in \mathbb{R}^n$$

- based on the *continuum hypothesis*
- give global description of spatio-temporal evolution
- good agreement with empirical data
- suitable for posing control and optimization problems

BUT : no general analytical theory for

- multi-D hyperbolic systems ($n > 1$)
- control of conservation laws
- able to recover complexity features of crowd dynamics?

Multi-scale approach

Micro-to-macro scaling:

- interactions among individuals at *microscopic* scale
- emergence of *self-organized* flow patterns at *macroscopic* scale

Outer-to-inner scaling:

- local *macroscopic* distribution of the crowd can modify interaction rules
- non-linearly additive *microscopic* pedestrian interactions

Multi-scale time-evolving probability measures

Probability distribution of pedestrians

$$\mu_t = \theta m_t + (1-\theta) M_t \quad \text{where} \begin{cases} m_t = \sum_{j=1}^{N} \delta_{P_j(t)} & \text{microscopic mass} \\ dM_t(\mathbf{x}) = \rho(t,\mathbf{x}) d\mathbf{x} & \text{macroscopic mass} \end{cases}$$

Governing equation: probability conservation deduced from individual-based modeling

$$\partial_t \mu_t + \nabla \cdot (\mu_t \vec{V}_t) = 0$$

$$\vec{V}_t(\mathbf{x}) = \vec{V}_d(\mathbf{x}) + N \int_{\mathcal{B}_R(\mathbf{x})} K(\mathbf{x},\mathbf{y}) d\mu_t(\mathbf{y})$$

(Cristiani-Piccoli-Tosin, 2011)

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(Cristiani-Piccoli-Tosin, 2011)

Dynamics at bottlenecks

Two groups with opposite directions passing through a door:

- $\theta = 0$: only macro
- $\theta = 1$: only micro
- $\theta = 0.3$: micro-macro

(Cristiani-Piccoli-Tosin, 2011)

Dynamics at intersections

Two groups with perpendicular directions crossing each other:

- $\theta = 0$: only macro
- $\theta = 1$: only micro
- micro-macro

(Cristiani-Piccoli-Tosin, 2011)

Concluding remarks

Models should:

- reproduce (qualitatively) emerging phenomena observed in real situations
- account for individual choices that may affect the whole system
- display few parameters to be identified by experiments
- be validated on empirical data

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Much still have to be done!

Thank you for your attention!