The model-based sparse component analysis framework incorporates the prior information on structured sparsity models (Chapter 4) and characterization of the acoustic multipath projections (Chapter 5) to obtain the best estimate of the spatio-spectral components matching the microphone array observations. Therefore, the model-based sparse recovery algorithms perform optimization in the observation space. Alternative to this objective, we can optimize the prediction error of the signal which has been the fundamental concept of spatial filtering techniques. Hence, the goal of this chapter is to incorporate the prior information on structured sparsity models and multipath projections to yield an optimum beamforming formulation.

8.1 A Multipath Sparse Beamforming Method

The model-based sparse component analysis framework stated in Chapter 3 assumes a discrete Euclidean geometry for source locations. We consider similar set-up to derive the formulation of beamforming for spatio-spectral information recovery. The objective is to characterize an optimum beamformer which takes into account the acoustic multipath. Recall from Chapter 3 that the microphones observation vector can be expressed as $\mathbf{X} = \Phi \mathbf{s}$ while $\Phi \in \mathbb{C}^{M \times G}$ denotes the array manifold matrix obtained from the generative forward model of the acoustic multipath. For the sake of brevity, we have omitted the frequency index. We will adopt this convention whenever no confusion can arise. The sparse recovery approach is a non-linear framework for joint localization and speech recovery. In this section, we elucidate how spatial filtering can be formulated to achieve the similar objective while taking into account the multipath acoustic. There are several alternatives for designing the beamformers. We focus our attention on minimum variance unbiased estimate of the signal using MVDR as well as the minimum mean square error beamformer. The former serves as a key component in various beamforming structures whereas the later formulates the optimum signal estimation. We assume that the noise is a sample function of a random process with known second-order statistics and it has similar characteristics at all sensors. In the subsequent sections, we derive the formulation to recover a single desired source signal [Asaei et al., 2013c]. This framework can be easily generalized to multiple source signals.
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8.1.1 Minimum Variance Distortionless Response Beamformer

The conventional MVDR beamformer performs weighted combination of the microphone recordings to enable separation of the signals and interference on the basis of their spatial characteristics. The spatial filtering operation generates the output as \( Y = W^H \mathcal{X} \) where \( ^H \) stands for conjugate transpose; the unknown filter weights \( W \in \mathbb{C}^{M \times 1} \) are optimized in order to minimize the overall noise and interference power. To guarantee that the signal coming from the desired direction is received without distortion, the optimization is performed subject to the distortionless constraint as follows

\[
W_{\text{MVDR}}^H = \underset{W}{\operatorname{argmin}} \mathbb{E} \{ W^H \mathcal{X} \}, \quad \text{s.t.} \quad W^H d = 1 \quad (8.1)
\]

where \( \mathbb{E} \) denotes the expectation operation and \( d \) is the steering vector for a plane wave coming from the desired direction. The solution to this criterion using a Lagrange multiplier is given by [Trees, 2002]

\[
W_{\text{MVDR}}^H = \frac{d^H R_{\mathcal{X}^*}^{-1}}{d^H R_{\mathcal{X}^*}^{-1} d} \quad (8.2)
\]

where \( R_{\mathcal{X}^*} \) denotes \( \mathbb{E} \mathcal{X} \mathcal{X}^H \). The criterion is also known as minimum power distortionless (MPDR) beamformer. In the original MVDR formulation, \( R_n \) is replaced with \( R_{\mathcal{X}^*} \), assuming that there are no model-errors, both criteria coincide so we refer to this criteria as MVDR. If \( R_n \) is available, the MVDR beamformer offers significant robustness to errors in steering vectors (direction of the desired source) estimation [Ehrenberg et al., 2010]. The MVDR beamformer yields a Maximum Likelihood (ML) solution assuming that the frequencies are independent and the signal wavenumber (or direction of arrival (DOA)) is known. However, the MVDR filter scanned in wavenumber space in most cases does not provide the ML estimate of the signal’s DOA [Trees, 2002].

Acoustic-Informed Sparse MVDR Beamformer

The conventional beamformer assumes that direction of the desired source is known. To incorporate the multipath effect, we assume that the acoustic channel corresponding to the desired source \(^1\) is known and denoted by \( \phi_s \). Hence, a Multipath MVDR (M-MVDR) beamformer is expressed as

\[
W_{\text{M-MVDR}}^H = \underset{W}{\operatorname{argmin}} \{ W^H R_{\mathcal{X}^*} W \} \quad \text{s.t.} \quad W^H \phi_s = 1, \quad (8.3)
\]

---

1. Room impulse response functions between the source and microphone array
The weights of M-MVDR are optimized such that acquisition of the signal with respect to the desired channel is distortionless; using a Lagrange multiplier, the optimal weights are given by

$$W_{M-MVDR}^H = \frac{\phi_s^H R_x^{-1} \phi_s}{\phi_s^H R_x^{-1} \phi_s}$$ (8.4)

Given the forward model of the reverberant acoustic characterized as $\Phi$, an additional constraint on the weights of an acoustic-informed beamformer is suggested as

$$W^H(\Phi B) = 1$$ (8.5)

where $B$ is a binary sparse vector whose support corresponds to the source location and selects the acoustic projection affecting the desired source. Therefore, estimation of $W^H$ amounts to estimating the vector $B$. If we ignore the amplitude distortion due to multipath propagation, estimation of the filter weights is achieved by the following optimization

$$\hat{B} = \arg\min_B \{B^H \Phi^H R_x \Phi B\} \quad \text{s.t.} \quad B^H \Phi^H \phi_s = 1$$ (8.6)

where $\phi_s$ denotes the multi-path channel corresponding to the desired source. We now solve the problem by imposing the constraint using a Lagrange multiplier. The function that we minimize is

$$F = B^H \Phi^H R_x \Phi B + \lambda[B^H \Phi^H \phi_s - 1] + \lambda^H[\phi_s^H W - 1]$$ (8.7)

Taking the complex gradient with respect to $B^H$ and solving (8.7) gives

$$B = -\lambda \frac{\Phi^H \phi_s}{\phi_s^H \Phi^H \phi_s}$$ (8.8)

To evaluate $\lambda$, we use the distortionless constraint and obtain

$$\lambda = -\frac{\Phi^H R_x \Phi}{\phi_s^H \Phi \Phi^H \phi_s}$$ (8.9)

Therefor, the multiparty MVDR (M-MVDR) solution is obtained as

$$\hat{B} = \frac{\Phi^H \phi_s}{\phi_s^H \Phi \Phi^H \phi_s}, \quad \hat{W}_{M-MVDR} = \frac{\Phi \Phi^H \phi_s}{\phi_s^H \Phi \Phi^H \phi_s}$$ (8.10)

Moreover, we can incorporate the prior information on sparse structure of $B$ and regularize the optimization stated in (8.6) by the $\ell_1$-norm of $B$ expressed as follows

$$\hat{B} = \arg\min_B \{||B^H \Phi^H \phi_s ||_2 + \lambda ||B||_1 \} \quad \text{s.t.} \quad B^H \Phi^H \phi_s = 1$$ (8.11)

The solution can be obtained by sparse recovery algorithm. The formulation of (8.11) enables better null steering so it offers a more robust solution when there are point interferences while
Chapter 8. Optimum Spatial Filtering

requiring fewer parameters to describe the solution than the conventional beamforming techniques. This formulation can be extended for diffuse noise field to obtain an acoustic-informed sparse superdirective beamformer which is an effective methodology for speech applications [Wolfel and McDonough, 2009].

8.1.2 Minimum Mean-Square Error Estimator

Alternative to MVDR, the linear array processor can be derived to estimate the signal using MMSE criterion. The weights of an MMSE beamformer are adapted to the reference signal \( S \) and obtained to minimize the average power in signal recovery error stated precisely as

\[
W_{\text{MMSE}}^H = \arg \min_{W^H} \mathbb{E} \{ |S - W^H A|^2 \},
\]

\[
= \arg \min_{W^H} \mathbb{E} \{ SS^H - W^H A S^H - S A^H W + W^H A A^H W \}. \tag{8.12}
\]

To find the minimum of this function, we take its derivative with respect to \( W^H \) and set the results equal to zero, thus we obtain

\[
W_{\text{MMSE}}^H = R_{S A} R_{A}^{-1}
\]

where \( R_{S A} = \mathbb{E} \{ S A^H \} \). Given \( A = d S + N \) where \( d \) denotes the steering vector and having the uncorrelated assumption between signal \( S \) and noise \( N \), the cross-correlation and auto-correlation matrices are

\[
R_{S A} = R_S d^H, \quad R_A = R_S d d^H + R_n \tag{8.14}
\]

This formulation requires acoustic and configuration stationarity assumption to obtain a reasonable estimate of the covariance of signal and noise. Using the matrix inversion formula to invert \( R_A \), we have

\[
R_A^{-1} = R_n^{-1} - R_n^{-1} R_S d (1 + d^H R_n^{-1} R_S d)^{-1} d^H R_n^{-1} \tag{8.15}
\]

hence, the following solution is obtained [Trees, 2002]

\[
W_{\text{MMSE}}^H = \frac{d^H R_n^{-1}}{d^H R_n^{-1} d} \cdot \frac{R_S}{R_S + d^H R_n^{-1} d} W_{\text{MVDR}}^{\text{Wiener post filter}}
\]

Hence, the MMSE estimator is a shrinkage of the MVDR beamformer which suppresses the interfering signals and noise without distorting the signal propagating along the desired source direction followed by a single-channel Wiener post-filter to yield the optimal signal estimation given the second-order statistics of the noise [Trees, 2002].
8.1. A Multipath Sparse Beamforming Method

Acoustic-Informed Sparse MMSE Beamformer

Given the forward model characterization of the reverberant acoustic, a multipath beamformer can incorporate an additional constraint expressed as $W^H(\Phi B) = 1$ so estimating the beamformer weights can be obtained through the following optimization

$$\hat{B}^H = \arg\min_{B^H} \{ ||S - B^H \Phi^H \mathcal{X} ||^2 \},$$

$$= \arg\min_{B^H} \{ R_S - B^H \Phi^H R_{\mathcal{X}^H} - R_{S,\mathcal{X}^H} \Phi^H B + B^H \Phi^H R_{\mathcal{X}^H} \Phi B \},$$

(8.17)

where the solution is obtained by equating the derivative with respect to $B^H$ to zero. Given $\mathcal{X} = \phi_s S + N$ and assuming that signal and noise are uncorrelated, the cross-correlation and auto-correlation matrices are

$$R_{S,\mathcal{X}^H} = R_S \phi_s^H, \quad R_{\mathcal{X}} = R_S \phi_s \phi_s^H + R_n$$

(8.18)

Hence, the multipath MMSE beamformer is obtained as

$$\hat{B}^H = \frac{R_{S,\mathcal{X}^H} \Phi}{\Phi^H R_{\mathcal{X}} \Phi}, \quad W^H_{\text{M-MMSE}} = \frac{R_S \phi_s \Phi^H}{\Phi^H (R_S \phi_s \phi_s^H + R_n) \Phi}$$

(8.19)

where $\phi_s$ denotes the acoustic channel corresponding to the desired signal.

Moreover, we can incorporate the sparsity constraint in formulation of a multipath sparse MMSE estimator obtained through the following optimization stated in (8.20); the distortionless constraint prevents the trivial zero solution. The sparse MMSE beamformer would then be

$$\hat{S}, \hat{B} = \arg\min_{\hat{B}, \hat{S}} \{ ||B^H \Phi^H \mathcal{X} - S ||^2 + \lambda ||B ||_1 \quad \text{s.t.} \quad B^H \Phi^H \phi_s = 1 \}$$

(8.20)

This formulation can be extended to incorporate the structured sparsity models pertained to the representation of source signal $S$ as described in Chapters 4 and 6 [Asaei et al., 2012c, 2013a].

Acoustic Calibration Beamforming

We can calibrate the model parameters of an acoustic-informed beamformer using a known signal at a known location. Recall from Chapter 5 characterization of the compressive acoustic projections. We proposed a novel formulation of the reverberation model factorized as $\Phi = O P$ where $O \in \mathbb{C}^{M \times \mathcal{G}}$ is the free-space Green’s function matrix such that each $O_{ij}$ component indicates the sound propagation coefficients, i.e. the attenuation factors and the phase shift due to the direct path propagation of the sound source located at cell $i$ (on a $\mathcal{G}$-point grid of actual-virtual sources) and recorded at the $j$th microphone. Given the $\mathcal{G}$-cell discretization, $O$ is computed from the propagation formula stated in Equation (3.3) and it is equal to $\Phi$ when $R = 0$. The other term, $P \in \mathbb{R}^{\mathcal{G} \times \mathcal{G}}$ is the permutation matrix such that its $i$th column contains the absorption factors.
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of $G$ points on the grid of actual–virtual sources with respect to the reflection of the $t^{th}$ actual source. Since the Image model characterizes the actual–virtual source groups, each column $P_{i}$ is consequently supported only on the corresponding group $\Omega_i$ i.e., $\forall i \in \{1 \ldots G\}, \forall j \in \Omega_i, P_{j,i} = 0$. We can calibrate the acoustic model parameters through the optimization stated as

$$\hat{\gamma} = \arg\min_{\gamma} \{ ||Y^H O^H \gamma - S||_2 + \lambda ||\Sigma\|_F \},$$

(8.21)

where $Y = PB$ and $\Sigma = YSS^H Y^H$, $\mathcal{F}$ denotes either the joint sparsity norm, $||\cdot||_{1,2}$, or nuclear norm, $||\cdot||_*$, for structured sparsity or low-rank encoding [Asaei et al., 2012c, 2013a, Golbabaee and Vandergheynst]. The optimization objective formulated in (8.21), tunes the acoustic model parameters such that the expectation of the signal estimation error is minimized. Alternatively, we can derive a calibration beamformer as

$$\hat{W} = \arg\min_{W} \{ ||W \varphi - \Phi \mathcal{F}||_2 + \lambda \| \mathcal{F} \|_1 \},$$

(8.22)

where the calibration filter $W$ is learned to minimize the deviation from the forward model $\Phi B$.

The discerning reader may note that if the condition stated in (8.5) holds, i.e., $W^H (\Phi B) = 1$, the sparse MMSE estimator stated in (8.20) is equivalent to sparse recovery formulation expressed in Chapter 3 as

$$\hat{\mathcal{F}} = \arg\min_{\mathcal{F}} \| \mathcal{F} \|_1 \quad \text{s.t.} \quad \mathcal{X} = \Phi \mathcal{F}$$

(8.23)

In a general set-up however, the two sparse MMSE estimators, (8.20) and (8.23), have different objectives; the earlier optimizes the prediction error and the later optimizes the observation error. Hence, the two formulations may be regarded as alternative approaches with different solution space geometry to address the processing of multiparty recordings.

8.2 Experimental Analysis

The experimental analysis are carried out to study the performance of the proposed methods with two specific scopes presented in Section (8.2.1) on source localization and Section (8.2.2) on signal estimation.

8.2.1 Source Localization and Spatial Resolution

We start our experimental analysis with a computational study on microphone array beam-pattern and spatial resolution. The generic theory of compressive sensing enables quantitative assessment of the side-lobe level in terms of the number of sources, inter-element spacing, number of sensors and acoustic conditions [Carin, 2009]. The theory of the performance bounds was asserted in Chapter 3. We perform empirical demonstrations to study the performance of sparse recovery
8.2. Experimental Analysis

and compare it with beamforming. We consider an 8-channel microphone array. As stated in (3.10), to recover 2 sources, the coherence of the measurements is required to be less than 0.2. We calculate the coherence of the measurement matrix for a random linear array with inter-element spacing, equal to half of the signal wavelength. We assume that the sources are standing towards the array and start from a coarse angular division and increase the resolution below which the required value of the coherence is exceeded. The finest resolution is delineated as the beam-width [Carin et al., 2011]. This experiment is performed for anechoic as well as moderately reverberant acoustic conditions (room reverberation time is 270ms). In addition, we compute the half-power beam-width of the beamformer as the angular deviation upon which the output power is reduced by 3dB [Trees, 2002]. The results are listed in Table 8.1. The theoretical relationship between the MVDR beam-width and acoustic multipath is not explicitly defined in beamforming literature so we measured the reverberant beam-width empirically where the output power is reduced by half with respect to the look direction.

Table 8.1 – Calculated beam-width for sparse recovery and conventional beamforming

<table>
<thead>
<tr>
<th>Acoustic Condition</th>
<th>Sparse Recovery</th>
<th>Beamforming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anechoic</td>
<td>7°</td>
<td>12°</td>
</tr>
<tr>
<td>Reverberant</td>
<td>11°</td>
<td>30°</td>
</tr>
</tbody>
</table>

The results show that the resolution at which the two sources are guaranteed to be distinguished is higher for the sparse reconstruction framework compared to the conventional beamforming. The larger aperture-size and random layout of microphone array render the compressive projections less coherent and better performance guarantees in sparse recovery whereas the conventional beamforming are designed for uniform sampling with a small inter-element spacing less than half of the system wave-length to prevent spatial aliasing.

The resolution of source localization is further investigated in another set of experiments. We assume that the array broadside is discretized uniformly and matrix $\Phi$ consists of all steering vectors in the DOA range of $[-90^\circ, 0^\circ] \cup [0^\circ, 90^\circ]$ with a sampling interval of 5°. The signal impinges on a uniform linear array consisted of 8 microphones at inter-element spacing equal to half of the wavelength. Randomly generated white Gaussian noises are added to the microphone measurements at signal to noise ratio (SNR=10dB) for evaluations in noisy condition. The MVDR beamformer stated in (8.10) is implemented. The results are shown in Figures 8.1-8.3. We can observe that the formulation of conventional MVDR has more sensitivity to noise and can not distinguish the sources closely located to each other.

8.2.2 Speech Recovery Performance

The initial speech recovery experiments are carried out on synthetic data to obtain the empirical performance bounds in a controlled well-defined scenario. Furthermore, we perform real data
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evaluations on Multi-Channel Wall Street Journal (MC-WSJ) corpus [Lincoln et al., 2005].

Synthetic Data Evaluations

We conducted the experiments on two scenarios, reverberant and far-field set-up.

- The *reverberant* scenario is recorded using 8-channel circular microphone array with radius 10 cm and spatial resolution 50 cm within an enclosure of dimension $3 \times 3 \times 3$; RT$_{60} = 180$ ms. The signals are 1000 speech frames sampled at 8 kHz and analyzed using Han function of length 64 ms and the weights are estimated per 10 frames.

- The *far-field* scenario is recorded using 8-channel uniform array with inter-element spacing equal to half of the wavelength and the directional resolution is 5°. The signals are 1000 trails of random samples of length 100. However, only 10% of data samples are used to compute the statistics of the conventional beamformer.

The signal to noise ratio is 20 dB. We assume that the signal and noise samples are known so the only uncertainty is attributed to the number of reliable samples available for beamforming. The CVX package is used for sparse beamforming optimization [Grant and Boyd]. Table 8.2 summarizes the results. The sparse recovery approach identifies the support of sparse components (i.e., source locations) accurately and enables optimal recovery by inverse filtering as elaborated in Section 7.5 and the accuracy is bounded by the noise level.

Table 8.2 – RMSE of signal recovery. The numbers in parenthesis show the performance of conventional beamforming formulation without sparsity regularization. The multipath sparse MVDR beamformer expressed in (8.11) is compared with multipath MVDR beamformer expressed in (8.3). Similarly the multipath sparse MMSE defined in (8.20) is compared with multipath MMSE defined in (8.19).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Beamf.</th>
<th>1 Source</th>
<th>2 Sources</th>
<th>3 Sources</th>
<th>4 Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reverberant</td>
<td>MVDR 0.03 (0.52)</td>
<td>0.27 (0.54)</td>
<td>0.39 (0.83)</td>
<td>0.49 (0.84)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MMSE 0.66 (0.83)</td>
<td>0.70 (0.84)</td>
<td>0.73 (0.96)</td>
<td>0.75 (0.96)</td>
<td></td>
</tr>
<tr>
<td>Farfield</td>
<td>MVDR 0.10 (0.10)</td>
<td>0.10 (0.10)</td>
<td>0.17 (0.35)</td>
<td>0.16 (0.47)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MMSE 0.07 (0.54)</td>
<td>0.08 (0.55)</td>
<td>0.14 (0.57)</td>
<td>0.16 (0.59)</td>
<td></td>
</tr>
</tbody>
</table>

The differences between the performance of sparse beamforming and the conventional formulation show that the sparse beamformer requires fewer parameters to estimate the solution than the conventional beamforming techniques hence, it enables accurate estimation when the number of reliable samples are limited and suggests a framework for missing data beamforming. Additionally, it enables better null steering and offers a more robust solution in multiparty recordings and a better resolution if the sources are closely located. The results of conventional beamformers using only direction of the desired source in a reverberant acoustic were poor so they are not included here. We can calibrate the model parameters of an acoustic-informed beamformer using a known source signal at a given location as formulated in Section 8.1.2.
8.2. Experimental Analysis

Real Data Evaluations

The real data evaluations are based on a subset of Multi-Channel Wall Street Journal Audio-Visual (MC-WSJ-AV) corpus [Lincoln et al., 2005] used for PASCAL Speech Separation Challenge II [Himawan et al., 2005, McDonough et al., 2007]. The subset consists of two concurrent speakers who are simultaneously reading sentences form the Wall Street Journal and being recorded with 8-channel circular microphone array. The speakers are either seating or standing at 1.5 × 1.5 table and a circular microphone array with diameter 20 cm is located at the center of the table. The total number of utterances is 356 (or 178, respectively given that two sentences are read at a time). The geometrical set-up ground-truth are not provided and in particular the height information are missing hence, calculation of the room acoustic channel to perform inverse-filtering incur inaccuracies.

The speech recognition system used in the experiments is identical to the one in [Mahdian et al., 2012, McDonough et al., 2007], except that three passes are applied (instead of four). The first pass is unadapted speech recognition. In the second pass, unsupervised maximum likelihood linear regression (MLLR) feature space adaptation is applied and the third pass employs full MLLR adaptation. The estimated speaker positions are the same ones used in [Mahdian et al., 2012, McDonough et al., 2007] for superdirective and delay-and-sum beamforming. The room reverberation time is 0.7 s and the corpus is highly noisy.

In addition to the conventional beamforming, we employed sparse recovery to provide the exact location of the sources on a discretized grid [Asaei et al., 2012a]. The speech recognition results are presented in Figure 8.4. The results show that providing the accurate positioning obtained by sparse recovery (as illustrated through Figure 8.1-8.3) improves the performance of the conventional beamforming methods. Future experiments will consider the multipath sparse beamforming method to enable more effective signal recovery.

2. I would like to acknowledge Rahil Mahdian at Saarland University for running the speech recognition scripts on my recovered data. The WRR results after the second pass adaptation for ICA, delay-and-sum (DS) beamforming, superdirective (SD) beamforming are 15%, 16.54%, 25.15% respectively. We did not have access to the results of ICA after third pass adaptation. Given the improvement obtained for beamforming, we can expect that ICA would also improve roughly by 14% thus ≈29% is reported in the figure. The implementation of ICA is available at [Murata et al., 1998]
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Figure 8.4 – Word Recognition Rate (WRR) using independent component analysis (ICA), delay-and-sum beamformer (DS), superdirective beamformer (SD) and model-based sparse recovery incorporated for superdirective beamformer (MSR-SD). The benchmark WRR for the headset microphone is 76.55% and for the distant microphone is 0%.

8.3 Conclusions

In this chapter, a novel formulation of beamforming is proposed for acquisition of the signals in reverberant acoustic clutter of interferences and noise. We derived the beamforming methods which incorporate the sparsity structure pertained to the acoustic source distribution and multipath propagation. The quantitative assessments demonstrate that sparse beamforming enables effective beampattern steering from far fewer samples than the conventional beamformers. In addition, linear constraint on the desired channel rather than the desired direction improves the signal estimation performance in reverberant enclosures. The experimental analysis in terms of source localization and speech recovery provides strong evidence of the effectiveness of sparsity models to enable high resolution source localization and improving the speech recovery performance.
8.3. Conclusions

Figure 8.1 – Sparse recovery vs. MVDR beamformer for localization of one source in noisy and clean condition; The SNR of noisy scenario is 10dB. The x-axis demonstrates the direction bins with a resolution of 5° and y-axis demonstrates the energy of the estimated signal in that corresponding direction.

Figure 8.2 – Sparse recovery vs. MVDR beamforming for localization of two sources in clean and noisy condition; the support of the sparse components is exact. The x-axis demonstrates the direction bins with a resolution of 5° and y-axis demonstrates the energy of the estimated signal in that corresponding direction.

Figure 8.3 – Sparse recovery vs. MVDR beamforming for localization of four sources in noisy condition; the support of the sparse components is exact. The x-axis demonstrates the direction bins with a resolution of 5° and y-axis demonstrates the energy of the estimated signal in that corresponding direction.
Bibliography


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